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Localizability and Coupling Constants

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Let us consider for free elementary systems the postulates: (i) localizability of systems would not favor in a physical sense some inertial frames and that (ii) standard quantum mechanics (or, at least, a skeleton of it) applies for localizability. It is known that, at least for the lower values of spin, (i) and (ii) imply a unique solution of the localization problem for the no-interaction case. Extrapolating the analysis to the case when interactions are present, we offer arguments in favor of the *conjecture* that if elementary systems under interaction are localizable, then (i) and (ii) imply restrictions on the coupling constants, and probably their single-valuedness.

Let us consider elementary systems in quantum special relativity and assume that localizability in space-time makes sense for these systems (at least if the word *localizability* is used in a broad sense). The localization problem is that of obtaining a mathematical characterization and the physical interpretation of the entities concerning localizability (Kálnay, 1971a and references quoted there). An approach was proposed in Kálnay (1970) and further developed in Kálnay and Torres (1971, 1973, 1974) and in Kálnay (1971b, 1973). The idea essentially[1] was to know how much information concerning the localization problem could be deduced from the postulates that (i) localizability of an elementary system would not favor in a physical sense some inertial frames of reference and that (ii) standard quantum mechanics (or, at least, a skeleton of it) applies for localizability. This program was developed for free massless or massive elementary systems of spins 0 and 1/2 and for massless spin 1. Certainly, the restriction to free systems deprives the results of much (but not all) of their physical relevance, but the problem was difficult enough that it seemed reasonable to learn first how to deal with the free case and to reserve for later research the case of interactions.

[1]Here the word "essentially" means that we are entering neither into the details nor into the technicalities of the assumptions.

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The present note concerns the case in which interactions are present and is based on a revision we did of the above papers. One would expect that (i) and (ii) impose some constraints on the notion of quantum localizability, but that assumptions (i) and (ii) do not give enough support to a full solution of the problem. Things should be even worse since in Kálnay (1970) the postulates were relaxed to a weak form. *Much to our surprise, (i) and (ii) led to a unique solution in all the cases studied.* As an example, let us consider the free massive spin-1/2 case. A component X^k of the position operator turns out to depend on functions of momentum p and on certain elements of the algebra of Dirac matrices; unknown coefficients appear in front of them; now, functions of p^r are specified if the values of the coefficients of power series developments are given; therefore, X^k depends on p, some Dirac matrices, and a set $\Gamma \equiv \{G_{(1)}, G_{(2)}, G_{(3)}, \ldots\}$ of unknown constants. In Kálnay (1970) we have shown that (i) and (ii) lead to precise values of $G_{(2)}, G_{(3)}, \ldots$ and, at that time, only the constant $G \equiv G_{(1)}$ remained unknown. [See equation (6.28) of the last reference.] In Kálnay and Torres (1974) we did a more careful analysis of the implications of the fact that the system has spin, and deduced from them that necessarily $G = 0$: *From the axioms (i) and (ii) it results that a solution exists and that it is unique. Each of the constants of the set* Γ *has only one allowed value.*

Let us now consider localizability of elementary systems interacting with themselves or with external fields. We are far from having solved the localization problem, but it is clear that *now* Γ *includes the coupling constants.* Could one again expect the miracle that all $G_{(n)}$ are single valued? We still cannot answer this question, but we believe that the above gives some support for proposing that interested people may add their own efforts to ours, in order to try to demonstrate the following hypothesis:

Conjecture. If elementary systems under interaction are localizable, then postulates (i) and (ii) imply restrictions on the values of the coupling constants, and probably their single-valuedness.

If the conjecture turns out to be correct, two interesting possibilities arise: (a) elementary systems are localizable, and then a method of computing (or of restricting the allowed values of) the coupling constants would have been found; or (b) this method leads, e.g., to a wrong value of the fine-structure constant, offering then a rigorous proof that realistic elementary systems are not localizable.

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